N-D Arrays

Arrays with more than 2-dimensions are referred to as
– "N-D arrays", or
– "multidimensional arrays"

Suppose we pictorially denote a 4-by-7 array as

Then a 4-by-7-by-6-by-5 array would be depicted as

Array Referencing

http://jagger.me.berkeley.edu/~pack/e177

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Full-Index Reference

Let A be a multidimensional array. Consider

\[ N = \text{ndims}(A); \quad \text{szA} = \text{size}(A) \]

LHS = expr involving \( A(\text{idx1}, \text{idx2}, \ldots, \text{idxM}) \)

If \( M = N \), then it is a **full-index reference**
- The \( \text{idxK} \) vector must be integer values between 1 and \( \text{szA}(K) \)
  - Row/column does not matter (here)
  - Actually, it can be an array, but Matlab reshapes it into a vector

What is \( \text{size}(A(\text{idx1}, \text{idx2}, \ldots, \text{idxM})) \)?

\[ \text{[length(idx1) length(idx2) \ldots length(idxM)]} \]

Standard rectangular row/column/page/book/… selection

\[ A = \text{rand}(4,7,6,5); \]
\[ B = A([1 2],[3 4 7],[1 3 6],[2 3 5]); \]

### Reduced Index Reference

Let A be a multidimensional array. Consider

\[ N = \text{ndims}(A); \quad \text{szA} = \text{size}(A) \]

LHS = expr involving \( A(\text{idx1}, \text{idx2}, \ldots, \text{idxM}) \)

If \( M < N \), then
- A **reduced-index reference** (if \( M = 1 \), single-index reference)
- For \( K \leq M \)
  - The \( \text{idxK} \) vector must have values between 1 and \( \text{szA}(K) \)
  - \( \text{idxK} \) acts as a reference into the \( K \)th dimension
- \( \text{idxM} \) must contain integers, between 1 and \( \prod(\text{szA}(M+1:N)) \)
- \( \text{idxM} \) acts as a linear order reference into dimensions
  \[ M, M+1, \ldots, N-1, N \]

What is \( \text{size}(A(\text{idx1}, \text{idx2}, \ldots, \text{idxM})) \)?

\[ \text{[length(idx1) length(idx2) \ldots length(idxM)]} \]

Multidimensional Referencing with : 

Let A be a multidimensional array. Consider

\[ N = \text{ndims}(A); \quad \text{szA} = \text{size}(A) \]

LHS = expr involving \( A(\text{idx1}, \text{idx2}, \ldots, \text{idxM}) \)

Suppose idxK is a single : (colon)
- If \( K = M \) and \( M = N \) (last index of full-index reference), then
  - The colon is expanded to mean \( (1: \text{szA}(N))' \), as expected
- If \( K < M \) (ie., not the last index), then
  - The colon is expanded to mean \( (1: \text{szA}(K))' \)
- If \( K = M \), and \( M < N \) (last index of reduced-index reference), then
  - The colon is expanded to \( (1: \prod(\text{szA}(K:N)))' \)

**Note:** The ‘’ is actually only important in the single-index

\[ A = \text{rand}(4,3) \]
\[ A(:) \quad \% 12-by-1 \]
\[ A(1:12) \quad \% 1-by-12 \]

Single-Index Reference

Let A be a multidimensional array with \( N = \text{ndims}(A); \)
\( \text{szA} = \text{size}(A), \) and a RHS expr involving \( A(\text{idx1}). \)

Then
- \( \text{idx1} \) must contain integers, between 1 and \( \text{prod(szA)} \)
- If \( \text{idx1} \) is a scalar, then \( A(\text{idx1}) \) is a scalar, namely that element of the array A, using “linear order reference”
- Generally, the value of \( B = A(\text{idx1}) \) satisfies
  \[ \text{size}(B) = \text{size}(\text{idx1}), \quad \text{and} \]
  \[ B(k) = A(\text{idx1}(k)) \]
  for each \( k \)
- Unless \( A \) has only 1 nonsingleton dimension and \( \text{idx1} \) is a row/column, then \( B \) will have the same singleton dimensions as \( A \)

Examples
- scalar, so linear order reference

\[ A = \text{rand}(3,4); \]
\[ A([1 8 11]), \quad A([1 8 11]’) \]
\[ \text{idx2} = \text{cat}([1 8 11],[2 9 10],3); \quad \% 1-by-3-by-2 \]
\[ A(\text{idx2}) \]

Reduced Index Reference

Example

\[ A = \text{rand}(6,5,4,3,2); \quad \% 6-by-5-by-4-by-3-by-2 \]
\[ A(1:4,3:5,10) \quad \% 4-by-3 \]

Here,
- 1:4 references 4 (of 6) rows in A
- 3:5 references 3 (of 5) columns in A
- 10 references, using linear ordering, the 2,3,1 entry of the remaining 4-by-3-by-2 dimensions

Alternatively, consider

\[ A(1:2,3:4,[1 4],[6 4 1]) \quad \% 2-by-2-by-3 \]

\[ A(2,3:end,4) \quad \% A(2,3:5,4), \quad 1-by-3-by-2 \]

Multidimensional Referencing with end

Same setup again

Suppose idxK uses the keyword end
- If \( K = M \) and \( M = N \) (last index of full-index reference), then
  - end is expanded to mean \( \text{szA}(N) \), as expected
- If \( K < M \) (ie., not the last index), then
  - end is expanded to mean \( \text{szA}(K) \), as expected
- If \( K = M \), and \( M < N \) (last index of reduced-index reference), then
  - end is expanded to \( \text{prod(szA}(K:N)) \)

Examples
- A = randn(4,5,6);
  A(end) \quad \% same as A(120)
  A(2:4,end-1:end) \quad \% same as A(2:4,[29 30])
  A(2,3,end,4) \quad \% A(2,3,5,4), \quad 1-by-3-by-1 (1-by-3)