Abstract

Analysis techniques currently being used for current military aircraft, e.g. F/A-22 and JSF aircraft, are well suited to certify flight control systems. These techniques though are complex, time consuming and require many well trained control and simulation engineers to accomplish. A high percentage of the future UAVs will be autonomous, capable of highly aggressive maneuvers using nonlinear, adaptive, reconfigurable flight control systems. Certification of these control algorithms will be even more of a challenge than current military aircraft and will require advances in nonlinear analysis specifically tailored for the flight clearance problem. In addition, the rapid evolution of uninhabited aerial vehicles (UAVs) will require certification of the flight control law be performed with fewer resources.

The Air Force recently adopted the “Pathfinder” initiative, which uses a spiral development process to incrementally improve the capability of a system. Under this new initiative for UAV development, flight control laws can be upgraded at every spiral. The need for a systematic and rapid tool for flight control law certification becomes more pressing as each spiral would only be 1-2 years, instead of the traditional 15-20 year development cycle. Moreover, when there is an sudden need to reconfigure external stores for existing UAVs arising from new operational scenarios, the tool can act as a first-cut tool in determining the new flight envelope for the UAV with the new store. We already have seen such evolving operational scenarios happening when the Predator was converted from a reconnaissance UAV into a multi-role weapons platform with the addition of Hellfire missiles.
The proposed research program focuses on nonlinear robustness analysis for flight control certification. The techniques to be applied include structured singular value and worst-case analysis methods, guided Monte Carlo and worst-case simulation [1], Integral Quadratic Constraints (IQCs) [2, 3], hybrid system analysis, sum-of-squares (SOS) methods [4] combined with nonlinear numerical optimization. The goal is to develop a flexible, comprehensive nonlinear robustness analysis package oriented towards flight control law certification. These tools will be made available to the aerospace community, practitioners and researchers alike.

Robustness analysis tools for nonlinear systems is not as well developed as that for linear systems. Some progress has been made in extending certain linear analysis techniques to nonlinear systems and in particular, conditions for robust stability and robust performance similar to the structured singular value have been established. For example, robustness of nonlinear systems has been shown to be equivalent to the existence of solutions to Hamilton-Jacobi equations or nonlinear matrix inequalities [5, 6]. However, computational methods to establish the existence of these solutions have not yet been developed to a level comparable to their linear counterparts. Our work will be tailored to specific flight control analysis problems and systems, drawing heavily on analysis techniques involving IQCs that has been funded by AFOSR [2, 3].

As an elementary example of the types of answers we seek, [4],[7], consider a system of the form
\[ \dot{x} = f(x) + g_w(x)w \] (1)
with \( x(t) \in \mathbb{R}^n \), \( w(t) \in \mathbb{R}^{n_w} \), \( f \in \mathcal{R}^n \), \( f(0) = 0 \), and \( g_w \in \mathcal{R}^{n \times n_w} \). Suppose we want to compute a bound on the set of points \( x(T) \) that are reachable from \( x(0) = 0 \) under (1), provided the disturbance satisfies \( \int_0^T w(t)^*w(t) \, dt \leq R \), \( T \geq 0 \).

Certainly if a continuously differentiable \( V \) satisfies
\[ V(x) > 0 \text{ for all } x \in \mathbb{R}^n \setminus \{0\} \text{ with } V(0) = 0, \text{ and} \] (2)
\[ \frac{\partial V}{\partial x}(f(x) + g_w(x)w) \leq w^*w \quad \forall x \in \{ x \mid V(x) \leq 1 \}, \forall w \in \mathbb{R}^{n_w}, \] (3)
then \( \{ x \mid V(x) \leq 1 \} \) contains the set of points \( x(T) \) that are reachable from \( x(0) = 0 \) for any \( w \) such that \( \int_0^T w(t)^*w(t) \, dt \leq 1 \), \( T \geq 0 \). If a “target” level set shape is defined by \( P_\beta := \{ x \in \mathbb{R}^n \mid p(x) \leq \beta \} \), for some given positive definite function \( p \), then we can minimize \( \beta \) such that that \( \{ x \mid V(x) \leq 1 \} \subset P_\beta \), giving the tightest “radius” in the shape...
defined by level sets of $p$. This leads to the conditions

$$\min_V \beta$$
such that

$$\{ x \in \mathbb{R}^n | V(x) \leq 0, x \neq 0 \} \text{ is empty},$$

(4)

$$\{ x \in \mathbb{R}^n | V(x) \leq 1, p(x) \geq \beta, p(x) \neq \beta \} \text{ is empty},$$

(5)

$$\left\{ \begin{array}{l} x \in \mathbb{R}^n, \\
    w \in \mathbb{R}^nw \\
    \frac{\partial V}{\partial x}(f(x) + g_w(x)w) \geq w^*w, \\
    \frac{\partial V}{\partial x}(f(x) + g_w(x)w) \neq w^*w \end{array} \right\} \text{ is empty}. \quad (6)$$

One method to verify the emptiness of these sets for a given $V$ and $\beta$ is the positivestellen-satz (P-satz). Relaxing the exact P-satz conditions results in sum-of-squares feasability problems, and optimizing these over $V$ typically requires an iteration (due to the bilinear nature of the conditions). Appropriate scaling gives bounds for $\int_0^T w(t)^*w(t)\,dt \leq R$ (as opposed to 1).

Conversely, lower bounds can be computed by explicitly searching over the signal set $w$. For example, elementary modifications to the algorithm in [1] make it suitable for the reachable set bounds, often leading to $w$ trajectories which satisfy the first-order conditions for optimality.

In terms of a specific example, consider the system

$$\begin{align*}
\dot{x}_1 &= -x_1 + x_2 - x_1x_2^2 \\
\dot{x}_2 &= -x_2 - x_1^2x_2 + w
\end{align*}$$

with $p(x) := 8x_1^2 - 8x_1x_2 + 4x_2^2$. Employing the conditions above, using a 4th order $V$ and simplifications to the P-satz conditions yields Figure 1 below, which shows the result of these calculations for many different values of $R$. The top dashed line is an upper bound on the reachable “radius” (as measured by $p$) of the state due to disturbances $w$. Associated with each point on this curve is a Lyapunov function and multipliers which certify its correctness (as an upper bound). Each point on the the line marked “Lower Bound” is associated with a specific signal $w$ that certifies its correctness (as a lower bound). The dotted line is the correct value (upper and lower bound equal) for the Jacobian linearization of the original system at $x = 0, w = 0$. As expected, the nonlinear system behaves (modestly in this case) differently from the linearization, for “large” inputs.

The offending signal $w$ (for $R = 6$, for example) is shown in Figure 2 along with the resultant response of $p(x)$. The peak value achieved by $p(x)$ is a point on the lower bound curve, and the dashed line at 7.6 is a guaranteed upper bound on the response.

In conclusion, the worst-case reachable values of $p(x)$ over square-integrable disturbances are known rather tightly, given the small gap between the inner (i.e., lower) and outer (i.e., upper) bounds.
Figure 1: Relationship between norm of disturbance and radius of reachable set

Figure 2: Worst-case (lower bound) \( w \) for \( R = 6 \), and associated trace of \( p(x(t)) \)
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References


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